

**4th ANNUAL SHEPHERD OPEN MATHEMATICS CONTEST, 2009**

**I** a) Consider the following equation:

$$|a - |b - |c - |d - |x |||| = 0$$

What conditions must  $a$ ,  $b$ ,  $c$  and  $d$  satisfy so that the equation has solutions ?  
In case these conditions are satisfied, find all solutions.

b) What is the maximal possible number of solutions ?

c) If the number of solutions is maximal, and  $a$ ,  $b$ ,  $c$ ,  $d$  are all positive integers, what is the smallest value of the largest of all solutions ? Can you generalize ?

**II** Vertex **A** of the rectangle **ABCD** is fixed. Vertices **B** and **C** ( not belonging to the same side ) lie on two given mutually perpendicular lines,  $l_1$  and  $l_2$ , intersecting in the point **O**. Find the *locus* of the 4th vertex **D** ! ( Locus means all possible locations that the point **D** can occupy. ) Justify your answer !

Hint: draw an accurate figure, and experiment.

**III** There are 25 apples on a table. Two people play a game by taking turns: at each turn one player takes away 1, 2 or 3 apples ( each player decides whether he/she wishes to take 1, 2 or 3 apples ). The one who takes the last apple, wins. Does either player have a winning strategy ? If yes, what is it, if not, why not ?

Hint: what happens if there are only 4 apples on the table ?